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Review Article

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Unitarity of the Leptonic Mass Matrix

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Abstract
The experimental data gathered to date have provided evidence for massive neutrinos that mix. The Cabibbo Kobayashi Maskawa quark mixing matrix is a unitary matrix that contains information on the strength of the flavour-changing weak interaction. It can be parameterized by three mixing angles and a CP-violating phase. Similarly one can build a 3x3 leptonic mixing matrix,called the Pontecorvo Maki Nakagawa Sakata matrix (PMNS matrix), to describe the standard neutrino-mixing paradigm. Its unitarity is a critical assumption. Neutrino oscillation experiments have improved and stabilised the precision on the measurements of the three angles and the CP violating phase, constraining strongly unitarity violation. In this work, we will use the most recent Nu-Fit of the PMNS parameters at in normal and inverted hierarchies to calculate the degree of precision of the unitarity of the matrix elements and the sensitivity in weak decays involving neutrinos.

Keywords: Neutrino Oscillation, PMNS Mixing Matrix, Weak Decays

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Introduction
The experimental observation of neutrino oscillations in solar, atmospheric, reactor and accelerator regimes have now provided evidence that neutrinos have mass and mix [1-8]. This has been theoretically predicted many years before experimental evidence [9-12].

KATRIN experiment performs precision spectroscopy of the tritium β-decay close to the kinematic endpoint. On the basis of the first five measurement campaigns, the experiment derived an upper limit of $m_\nu<0.45eV[95\%CL]$ [13].

In April 2024, the DESI collaboration presented the most stringent bound on the sum of neutrino masses within the standard cosmological model by combining its baryon acoustic oscillation data with CMB ones: $\sum m_\nu<0.072eV[95\%CL]$ [14].

Lower bounds derived from neutrino oscillations give [15]: $\sum m_\nu>0.057eV$ in normal ordering hierarchy and $\sum m_\nu>0.096eV$ in the inverted ordering hierarchy.

These lower and upper bounds are very close and seem to favor the normal hierarchy. Nevertheless, the cosmological neutrino mass constraints are substantially relaxed if the background dynamics are allowed to deviate from flat Λ CDM.

This implies unambiguously that the standard model (SM) of electroweak interactions has to be extended at least in the leptonic sector since neutrinos are massless and left handed.

We will assume three massive neutrinos and the desert hypothesis between the electroweak energy scale i.e. the vacuum expectation value of the Higgs field (v=246 GeV) and the scale at which the SM is no longer valid. This scale Λ corresponds to the loss of vacuum stability with a Higgs boson of m_H=125 GeV.

The renormalisation group equation for the Higgs self coupling λ gives:

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$$\lambda(\Lambda) = \lambda(v) + \frac{3}{16\pi^2 v^4}\left(-4m_t^4 + 2M_W^4 + M_Z^4\right)\ln\left(\frac{\Lambda^2}{v^2}\right) \quad (1)$$

where m_t=172.52 GeV is the top quark mass [16].

Since $\lambda(v) = \frac{m_H^2}{2v^2}$, one gets ,with m_H=125 GeV, the upper bound is obtained by requiring λ(Λ)>0. One gets:

$\Lambda < 1.7 10^{14}\ GeV$

Assuming no extra neutrino and no extra particle like leptonic doublet or Higgs up to Λ, the simplest dimension five operator generating Dirac neutrino masses is the Weinberg operator [17]. This operator is the unique gauge invariant dimension 5 operator we can build with SM fields that gives rise to neutrino masses. The mass term reads:

$$m_{\alpha\beta}^{\nu} = C^{\alpha\beta}\frac{v^2}{\Lambda} \quad (2)$$

where $C^{\alpha\beta}$ is a Wilson coefficient and indices refer to leptonic flavor. Taking $\Lambda=1.7 10^{14}$ GeV we get a mass of 0.35eV with $C^{\alpha\beta}$=1, which is consistent with experimental limits on neutrino masses. In other words, we can accomodate massive neutrinos with the desert hypothesis up to the scale where the stability of the vacuum in the SM is lost.

Neutrino oscillation studies are carried out assuming a unitary mixing matrix for the rotation between eigenstates of flavor and eigenstates of mass: the Pontecorvo Maki Nakagawa Sakata matrix U (PMNS matrix).

In this work, we will test the unitarity of this matrix from the most recent NU- Fit parameters [18]. In section 2, we will use the normal ordering and inverted ordering best fits in the 3σ range to evaluate $|U|$ and $|UU^{+}|$ matrix elements. In section 3, we will consider weak decays involving neutrinos which are also sensitive to the PMNS matrix.

Our results are summarized and conclusions are given in section 4.

Unitarity of PMNS Matrix

Parametrisation of the Mixing Matrix

The PMNS matrix U relates neutrino flavor eigenstates to mass eigenstates:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1}U_{e2}U_{e3} \\ U_{\mu 1}U_{\mu 2}U_{\mu 3} \\ U_{\tau 1}U_{\tau 2}U_{\tau 3} \end{pmatrix}\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

U can be parametrised as:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\ \exp(-i\delta CP) \\ -s_{12}c_{23}\ -c_{12}s_{23}s_{13}\ \exp(i\delta CP) & c12c23\ -s_{12}s_{23}s_{13}\ \exp(i\delta CP) & s_{23}c_{13} \\ s_{12}s_{23}\ -c_{12}c_{23}s_{13}\ \exp(i\delta CP) & -c^{12}s_{23}\ -s_{12}c_{23}s_{13}\ \exp(i\delta CP) & c_{23}c_{13} \end{pmatrix} \quad (3)$$

θ_{12}, θ_{23} and θ_{13} are the mixing angles and δ_{CP} the only physical phase if we assume that neutrinos are Dirac particles. Note that neutrino oscillations are not sensitive to additionnal Majorana phases.

A number of global fits have been performed, leading to relatively precise values of the mixing angles and of the phase. We will take the latest data from shown in the table NUFiT 6.0 (2024) below. More precisely, for our analysis we will take IC24 with Super-Kamiokande and IceCube atmospheric data (best fit 3σ) [18-21].

NuFIT 6.0 (2024)

IC19 without SK atmospheric data		Normal Ordering ($\Delta\chi^2 = 0.6$)		Inverted Ordering (best fit)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
	$\sin^2\theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.345$	$0.308^{+0.012}_{-0.011}$	$0.275 \to 0.345$
	$\theta_{12}/^\circ$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$
	$\sin^2\theta_{23}$	$0.561^{+0.012}_{-0.015}$	$0.430 \to 0.596$	$0.562^{+0.012}_{-0.015}$	$0.437 \to 0.597$
	$\theta_{23}/^\circ$	$48.5^{+0.7}_{-0.9}$	$41.0 \to 50.5$	$48.6^{+0.7}_{-0.9}$	$41.4 \to 50.6$
	$\sin^2\theta_{13}$	$0.02195^{+0.00054}_{-0.00058}$	$0.02023 \to 0.02376$	$0.02224^{+0.00056}_{-0.00057}$	$0.02053 \to 0.02397$
	$\theta_{13}/^\circ$	$8.52^{+0.11}_{-0.11}$	$8.18 \to 8.87$	$8.58^{+0.11}_{-0.11}$	$8.24 \to 8.91$
	$\delta_{\rm CP}/^\circ$	177^{+19}_{-20}	$96 \to 422$	285^{+25}_{-28}	$201 \to 348$
	$\frac{\Delta m^2_{21}}{10^{-5}\ {\rm eV}^2}$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ {\rm eV}^2}$	$+2.534^{+0.025}_{-0.023}$	$+2.463 \to +2.606$	$-2.510^{+0.024}_{-0.025}$	$-2.584 \to -2.438$

IC24 with SK atmospheric data	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.1$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.308^{+0.012}_{-0.011}$	$0.275 \to 0.345$	$0.308^{+0.012}_{-0.011}$	$0.275 \to 0.345$
$\theta_{12}/^\circ$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$
$\sin^2\theta_{23}$	$0.470^{+0.017}_{-0.013}$	$0.435 \to 0.585$	$0.550^{+0.012}_{-0.015}$	$0.440 \to 0.584$
$\theta_{23}/^\circ$	$43.3^{+1.0}_{-0.8}$	$41.3 \to 49.9$	$47.9^{+0.7}_{-0.9}$	$41.5 \to 49.8$
$\sin^2\theta_{13}$	$0.02215^{+0.00056}_{-0.00058}$	$0.02030 \to 0.02388$	$0.02231^{+0.00056}_{-0.00056}$	$0.02060 \to 0.02409$
$\theta_{13}/^\circ$	$8.56^{+0.11}_{-0.11}$	$8.19 \to 8.89$	$8.59^{+0.11}_{-0.11}$	$8.25 \to 8.93$
$\delta_{CP}/^\circ$	212^{+26}_{-41}	$124 \to 364$	274^{+22}_{-25}	$201 \to 335$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.513^{+0.021}_{-0.019}$	$+2.451 \to +2.578$	$-2.484^{+0.020}_{-0.020}$	$-2.547 \to -2.421$

The parameters θ_{12}, θ_{13}, Δm^2_{21} and Δm^2_{3l} are well-determined with relative precision at 3σ of about 13%, 8%, 15%, and 6%, respectively. The mixing angle θ_{23} still suffers from the octant ambiguity and the CP phase δ_{CP} depends on the neutrino mass ordering.

Estimate of Unitary from Neutrino Oscillations

The absolute values of the PMNS matrix elements $|U|$ are respectively for normal ordering (NO) and inverted ordering (IO) taking the minimal and maximal values of column 2 (3σ range):

$$|U|_{NO} = \begin{pmatrix} 0.799-0.825 & 0.519-0.581 & 0.142-0.154 \\ 0.355-0.473 & 0.452-0.668 & 0.646-0.755 \\ 0.369-0.411 & 0.531-0.677 & 0.636-0.745 \end{pmatrix} \quad (4)$$

and:

$$|U|_{IO} = \begin{pmatrix} 0.799-0.842 & 0.519-0.580 & 0.143-0.155 \\ 0.312-0.467 & 0.460-0.674 & 0.656-0.758 \\ 0.376-0.433 & 0.513-0.671 & 0.636-0.729 \end{pmatrix} \quad (5)$$

We estimate uncertainty in unitarity by computing $|UU^+|$:

$$|UU^+|_{NO} = \begin{pmatrix} 0.9984-0.9996 & 0.0007-0.001 & 0.0007-0.0086 \\ 0.0007-0.001 & 0.990-0.999 & 0.0005-0.0062 \\ 0.0007-0.0086 & 0.0005-0.0062 & 0.999-1.007 \end{pmatrix} \quad (6)$$

and:

$$|UU^+|_{IO} = \begin{pmatrix} 0.9996-0.9998 & 0.0002-0.0006 & 0.0001-0.0005 \\ 0.0002-0.0006 & 0.9824-1.005 & 0.0006-0.0031 \\ 0.0001-0.0005 & 0.0006-0.0031 & 0.982-0.998 \end{pmatrix} \quad (7)$$

If we take the best fit data (NO bfp $\pm 1\sigma$):

$$|UU^+|_{NO} = \begin{pmatrix} 0.998 & 0.0007 & 0.00007 \\ 0.0007 & 0.999 & 0.0005 \\ 0.00007 & 0.0005 & 0.999 \end{pmatrix} \quad (8)$$

Therefore the $|UU^+|$ elements agree with those expected in the unitary case, within a precision better than 1% in NO mass hierarchy and 2% in IO mass hierarchy.

Unitarity in the lepton sector is experimentally confirmed from 2024 data.

Unitary from Weak Decays Involving Neutrinos

As pointed in [22, 23], the radiative decays of charged leptons $((l,l')=(e,\mu),(e,\tau)$ or $(\mu,\tau))$ like $l' \to l\gamma$, $l' \to l\bar{\nu}_l \nu_{l'}$ or W and Z decays involving neutrinos could be another tool to exhibit deviations from unitarity.

The lagrangian for charged and neutral current interactions gets modified since the flavored neutrino $\nu_l = \sum_{i=1}^{3} U_{li}\nu_i$ If α, β and γ label different leptonic flavours, the ratios $R^{\alpha}_{\beta\gamma}$ are given by:

$$R^{\alpha}_{\beta\gamma} = \frac{\Gamma\left(l_\alpha \to \nu_\alpha \beta \bar{\nu}_\beta\right)}{\Gamma\left(l_\alpha \to \nu_\alpha \gamma \bar{\nu}_\gamma\right)} = \sqrt{\frac{(UU^+)_{\beta\beta}}{(UU^+)_{\gamma\gamma}}} \quad (9)$$

Similarly for W decays:

$$R^{W}_{\beta\gamma} = \frac{\Gamma\left(W \to \beta \bar{\nu}_\beta\right)}{\Gamma\left(W \to \gamma \bar{\nu}_\gamma\right)} = \sqrt{\frac{(UU^+)_{\beta\beta}}{(UU^+)_{\gamma\gamma}}} \quad (10)$$

Since the unitarity elements involved in these ratios, which correspond to diagonal terms of UU^+, are very close to one, the deviations from SM are too tiny to be observed experimentally.

Conclusion

The PMNS leptonic mixing matrix may be generically non unitary. It is important to know up to which point the values of the matrix elements are allowed by data to differ from those obtained in the usual unitary analysis, as putative windows of new physics. The up to date analysis of neutrino oscillations data we have performed don't establish first signs of non -unitarity of the PMNS mixing matrix. Similar conclusions hold for weak decays involving neutrinos.

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