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Review Article

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The Complete Solution of the Gasca-Maeztu Hypothesis

Dr. Slavik Avagyan

National Polytechnic University of Armenia , Gyumri Campus, Armenia

Corresponding author
Dr. Slavik Avagyan, National Polytechnic University of Armenia , Gyumri Campus, Armenia.

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ABSTRACT
The article provides a proof of the hypothesis for any n number of points and uses fundamentally new approaches to prove it. The first of these approaches is that the number of binary combinations from the specified n points is limited. That is, after removing any point, when the remaining points are covered by lines, overlapping lines appear, because the binary connections of the points are already repeated. And the second approach is that no more than (N+1) points can be located on straight lines satisfying the conditions of the hypothesis. N is the number of lines.

Keywords: Gasca-Maeztu, Hypothesis, Point, Affiliation, Direct, Binary Conjugations

Introduction

According to the Gaska-Maeztu hypothesis, if any (n-1)of the

$$n = \frac{(N+1)(N+2)}{2}$$

points are covered by N lines, then there will be (N+1) points on at least one of the lines.
This hypothesis was proposed more than 40 years ago [1], but until now its solution has been given for N≤5 cases.

In the Work, the Hypothesis is Proved, Based on the Following Four Foundations

- if any two points a and b simultaneously belong to two lines l_1 and l_2

$(a,b) \in l_1$ and $(a,b) \in l_2$ then lines l_1 and l_2 are the same line L.

- if, say, points a, b, c, d are on linel_1, and points a, b, g are on linel_2, then all a,b,c,d,g 5 mentioned points will be on the L line.

Consequence

The number of points on a straight- line L will be less than the sum of the points of the two lines, by the number of overlapping points.

- In the case of a problem with given conditions, each point must lie on at least two straight lines.

Lemma

According to the conditions of the Gasky-Maestu hypothesis, no more than (N+1) points can be found on any of the N lines covering any (n-1)points.

Proof

Suppose that of a_1, a_2, a_3 .... a_n points a_2, a_3 .... a_n (n-1) points (without a_1) are covered by N lines and are on the i-th line (N+2) points. If we need to consider other (n-1) points where, say, point a_2 does not exist and which belongs to the i -th line, then the remaining (N+1) points belonging to this line should be located on different (N+1) lines (according to premises I. and II.), which, however, contradicts the basic condition of the hypothesis.

Thus, one of the most important consequences is that on any line satisfying the conditions of the hypothesis, there can be no more than N+1 points.

Proof of the Hypothesis

So, we have

$$n = \frac{(N+1)(N+2)}{2} \quad (1)$$

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points, any (n-1) of which is covered by N lines.

The number of binary pairs of n points is equal to

$$C_{(n,2)} = \binom{n}{2} = \frac{n(n-1)}{2} \tag{2}$$

As an initial option, consider covering (n-1) points from a_n points a_1, a_2, a_3 .... a_n without a_1. In other words, N lines cover (n-1) points եւ a_2, a_3 .... a_n.

Suppose, in the basic version, point a_2 and a number of k points are located on the i-th line. Since any (n-1) points must be covered by N lines, therefore, if point a_2 is not taken into account, then k points on the same line with it must be rearranged on different lines in accordance with principles I. and II. Otherwise, if any two of these k points are rearranged on the same straight line, then this line will also pass through pointa$_2$, which is unacceptable.

And so, sequentially a_1, a_2, a_3 .... a_n points from (n-1) points should be covered with N lines without a_3, without a_4.... without a_n (the order is conditional)

For each unobservable point, the points on the same line with it must be rearranged into different lines. Such permutations lead to new binary pairs for points in each new line. And if there are binary pairs that already existed in any of the previous lines, then this line and the line with the same binary pairs will coincide.

Because the second basis will occur and straight lines will be obtained from the basic version, where the number of points will increase. Then a line with (N+1) points can be obtained. This actually completes the proof.

First, let's prove that after removing some points, covering all the remaining (n-1) points with N lines, in the case of further removed points, by choosing only from those lines, it will not be possible to cover the remaining (n-1) points corresponding to the next removed point.

Proof

Thus, let's define the lines covering the remaining points a_2, a_3 .... a_n without a_1-

$$L_{a_1}^1, L_{a_1}^2, L_{a_1}^3, ..., L_{a_1}^N \tag{3}$$

let us denote the lines covering the points a_1, a_3 .... a_n (n-1) with N lines without a_2-

$$L_{a_2}^1, L_{a_2}^2, L_{a_2}^3, ..., L_{a_2}^N \tag{4}$$

let us denote the lines covering the points $a_1, a_3, ... a_{i-1}, a_{i+1}$ .... a_n (n-1) with N lines without a_i-

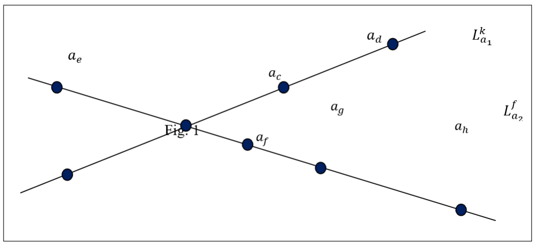
$$L_{a_i}^1, L_{a_i}^2, L_{a_i}^3, ..., L_{a_i}^N \tag{5}$$

For example, after a_1- and a_2-we remove say a_4-point.

Let's prove that without rearranging the points, we cannot cover the remaining (n-1) points with N lines by simply taking N lines from (3) and (4).

As we know, at least two straight lines must pass through point a_4.

Let's say: $L_{a_1}^k$ and $L_{a_2}^k$ (Figure 1) , k=1,2,3,…,N , f=1,2,3,…,N



a) first, we prove that we cannot use only the lines $L_{a_1}^1, L_{a_1}^2, L_{a_1}^3, ..., L_{a_1}^N$ that are obtained without the point a_1.

As can be seen from the figure, since the line $L_{a_1}^k$ passes through the point a_4, so another line must be introduced instead, so that the remaining *(n-1)* points are covered. However, this is not possible, because the points a_b, a_c, a_d must be located on different lines, therefore there will be new rearrangements of points, so new binary conjunctions will arise.

We can't use just the one without a_2 either.

$L_{a_2}^1, L_{a_2}^2, L_{a_2}^3, ..., L_{a_2}^N$ by removing $L_{a_2}^f$ and replacing it with lines covering the remaining (n-1) points.

Permutations are required, because the points a_e, a_f, a_g a_h on it must be on different lines, therefore, new binary connections will arise.

b) consider the case when, after removing point a_4, we can cover the remaining (n-1) points by selecting N lines from lines (3) and (4).

Suppose among those lines there are lines passing through points a_b, a_c, a_d, a_e, a_f, a_g a_h (for our case, they should be at least three).

Since we have to take these straight lines, the points connected to the remaining lines $L_{a_1}^1, L_{a_1}^2, L_{a_1}^3, ..., L_{a_1}^N$ and $L_{a_2}^1, L_{a_2}^2, L_{a_2}^3, ..., L_{a_2}^N$ should be rearranged to ensure that, except for point a_4, the remaining (n-1) points are covered by N lines.

In other words, even more binary pairs of points will appear.

Thus, after removing each point, new regroupings of points occur, and therefore new binary pairings.

Now let's determine the number of removed points, after which repetitions of binary pairs of points belonging to lines will occur. And this means that two straight lines on which the same binary parallels will be found will coincide and according to **principle II ,** the number of points of the common line will be equal to the set of points of those two straight lines.

After the initial version, when deleting a new point, we must choose such point rearrangements so that the following

$$C_{(n,2)} = \binom{n}{2} = \frac{n(n-1)}{2}$$

Binary pairs do not repeat

However, after the initial version, we will have (n-1) points that should also be removed from the initial version. In other words, no straight line should pass through them.

Now let's see how many binary combinations of points can occur after removing points
a_2, a_3 .... a_n from the initial version.
Thus according to (1) we have

$$n = \frac{(N+1)(N+2)}{2}$$

From here

$$n-1 = \frac{N^2+3N+2}{2} - 1 = \frac{N^2+3N}{2} \tag{6}$$

Since these points must be located on N lines, therefore the average for each line will be

$$n_{aver} = \frac{N^2+3N}{2N} = \frac{N}{2} + \frac{3}{2} \tag{7}$$

point.

The number of binary pairs of m points is equal

$$C_{(m,2)} = \binom{m}{2} = \frac{m(m-1)}{2}$$

therefore, on average, the number of binary pairs of points on each line will be equal

$$C_{(n_{avr},2)} = \binom{n_{avr}}{2} = \frac{\left(\frac{N}{2}+\frac{3}{2}\right)\left(\frac{N}{2}+\frac{1}{2}\right)}{2} \tag{8}$$

Since when removing a point from the original version of the location of points on N lines, the points on the same line with it must be rearranged on different lines, then the number of new lines will be equal

$$\frac{N}{2} + \frac{3}{2} - 1 = \frac{N}{2} + \frac{1}{2} \tag{9}$$

In other words, from the already existing lines, you can use

$$N - \left(\frac{N}{2} + \frac{1}{2}\right) = \frac{N}{2} - \frac{1}{2}$$

number of rows.

Thus, in case of rearrangement of the points related to the removed point on $\frac{N}{2}+\frac{1}{2}$ lines, taking into account (9), the number of binary conjunctions of these points will be equal to

$$C^2_{a_i^{as}} = \frac{1}{2}\left(\frac{N}{2}+\frac{3}{2}\right)\left(\frac{N}{2}+\frac{1}{2}\right)\left(\frac{N}{2}+\frac{1}{2}\right) \tag{10}$$

where a_i^{as} is the number of points associated with point a_i.
Now let's find the minimum number of points a_i to be removed, when the number of conjugations resulting from the rearrangements of the points connected with them will be equal to binary conjugations from n points

$$C_{(n,2)} = \binom{n}{2} = \frac{n(n-1)}{2}$$

to the number.

That is, after that, the binary pairs resulting from the rearrangement will be repeated on the new lines of the points connected to each removed point a_i. In that case, according to principle I, the lines on which these parallels are located will coincide and the number of points on the common line will be equal according to II. foundation.

Thus, the minimum number of points removed a_{min}, according to (2) and (10), will be equal to

$$a_{min}\frac{1}{2}\left(\frac{N}{2}+\frac{3}{2}\right)\left(\frac{N}{2}+\frac{1}{2}\right)^2 = \frac{n(n-1)}{2} \tag{11}$$

Using (6), we can write

$$a_{min}\frac{1}{2}\frac{(n-1)}{N}\left(\frac{N}{2}+\frac{1}{2}\right)^2 = \frac{n(n-1)}{2}$$

from wherever

$$a_{min} = \frac{4nN}{(N+1)^2} \tag{12}$$

Thus, by sequentially removing at least this number of points from the initial version, it turns out that when rearranging the points connected to the next and other removed points on other lines, the resulting binary conjugations will repeat the binary conjugations of the existing points.

That is, after sequentially removing

$$a_{min} = \frac{4nN}{(N+1)^2}$$

points from the base version, after removing the next point, matching lines will be obtained. The number of points on the common line will be equal to the sum of the points on those lines minus two.

For example

When N=5 n =21 it will be

$$a_{min} = \frac{4.21.5}{(5+1)^2} = 11,66$$

That is, after removing at least 12 points, there will be repeats from the binary pairs with n points.

When N=8 n =45 it will be

$$a_{min} = \frac{4.45.8}{(8+1)^2} = 17,77$$

That is, after removing at least 18 points, there will be repeats from the binary pairs with n points. As each successive point is removed, the repeated binary pairs of points will obviously increase rapidly.

Due to repeated binary conjunctions, according to premise I, some lines will coincide and the number of points on the new line will be (according to (7) and “Corollary”) equal.

$$2\left(\frac{N}{2} + \frac{3}{2}\right) - 2 = N + 1$$

Since the number of points is constant, it can be strictly asserted that if the number of points is less than N+1 when some two lines coincide, then there will necessarily be two coincident lines whose sum of points will be equal to N+1, but not more than that (according to “IV. Lemma”).

The hypothesis is proven.

Conclusion

Thus, Gasca-Maeztu’s hypothesis, which seemed unsolvable for more than forty years, was proved in the work. The proof of the hypothesis rests on four simple premises. And as the main condition is the limitation of binary combinations made of elements (points) and that no more than N+1 points can be found on any line [1].

The work has theoretical and applied significance.

Reference

1. Gasca M, Maeztu JI. [Numer. Maт. 39, 1-14 (1982; Zbl 0457.65004)]