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Short Communication

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Real Numbers of the Square Root of a Negative Number

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Annotation
The paper considers a special case when the result of extracting the square root from a negative number are real numbers.

Keywords: Negative Numbers, Square Root, Lorentz Transformation, Special Theory of Relativity.

It is well known that the square root of a positive number ($d > 0$) has two values with opposite signs, but equal in modulus.

$$d^{1/2} = \pm a \qquad (1)$$

It is clear that with the reverse operation, i.e. when squaring any real number, the result will always be a positive number.

$$a^2 = d \quad \text{and} \quad (-a)^2 = d \qquad (2)$$

Operation (2) can also be represented as multiplying a number by itself.

$$a\, a = d \quad \text{and} \quad (-a)(-a) = d \qquad (3)$$

From the last example, it can be seen that the opposite numbers ($+a$) and ($-a$) act as independent parameters that are not related to each other.

It is also known that the extraction of the square root from a negative number is impossible because the result of the inverse operation, i.e. the squaring of any real number will be a non-negative number (3).

And if in mathematics it is possible to find the root of a negative number by using an imaginary unit ($i^2 = -1$), then in physics the appearance in the corresponding formulas of the expression $[(-d)^{1/2}]$ is interpreted as the absence of the physical phenomenon itself, since such an equation in the real number system has no solution.

Meanwhile, in some cases, this general approach leads to misinterpretation of the results obtained.
Consider a system of linear equations:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + b_1 &= 0 \\ a_{21}x_1 + a_{22}x_2 + b_2 &= 0 \end{aligned} \qquad (4)$$

in which, for example, the following dependency exists:

$$a_{11}a_{22} = d \qquad (5)$$

at the same (d) can take both positive and negative values.

Suppose, according to the conditions of the problem for a special case, the values of the coefficients (a_{11}) and (a_{22}) should be equal to each other modulo.

$$|a_{11}|=|a_{22}| \qquad (6)$$

Then, for ($d > 0$), the values of one of the coefficients are defined as:

$$(a_{11})_{1,2} = \pm(d)^{1/2} \quad \text{and} \quad (a_{22})_{1,2} = \pm(d)^{1/2}$$

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As you can see, the results of these calculations are similar to operations (2) and (3). And here it is necessary to pay attention to the following circumstance. Based on requirement (5), the obtained values ($\pm a_{11}$) and ($\pm a_{22}$) in the reverse operation, as in equation (3), are multiplied by each other only with the same signs:

$(a_{11})_1\ (a_{22})_1 = (d)^{1/2}\ (d)^{1/2} = d$ or
$(a_{11})_2\ (a_{22})_2 = [-(d)^{1/2}]\ [-(d)^{1/2}] = d$ (7)

In the second case, at ($d < 0$), the reverse operation similar to (2) becomes impossible. Meanwhile, an operation similar to (3) exists, but is already performed with opposite numbers, i.e. equality (5) is always performed when in equation (7) coefficients (a_{11}) and (a_{22}) are applied with opposite signs, and then:

$(a_{11})_1\ (a_{22})_2 = (d)^{1/2}\ [-(d)^{1/2}] = (-d)$ or
$(a_{11})_2\ (a_{22})_1 = [-(d)^{1/2}]\ (d)^{1/2} = (-d)$ (8)

Thus, the peculiarity of the above special case is that the opposite numbers (8) are present in the equations simultaneously, since they are elements of the same system of equations (4), the parameters of which, according to the conditions of the problem (5), are directly related to each other.

In this case, the operation to extract the square root from a negative number will represent the following action:

$a_{11} = (-d)^{1/2} = \pm (|-d\,|)^{1/2}$, at the same time: $a_{22} = (-d)/a_{11}$

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}} \qquad x = \frac{x' + vt'}{\sqrt{1-\beta^2}}$$

$$y' = y \quad y = y' \qquad z' = z \quad z = z' \tag{9}$$

$$t' = \frac{t - \beta\frac{x}{c}}{\sqrt{1-\beta^2}} \qquad t = \frac{t' + \beta\frac{x'}{c}}{\sqrt{1-\beta^2}}$$

where: $\beta = v/c$

As is known, SRT imposes a ban on any movement with superluminal speeds, because it is believed that at ($v > c$) the expression $[(1-\beta^2)^{1/2}]$ becomes imaginary [1].

However, this conclusion is questionable, since equations (9) should be considered collectively, and not separately.

Meanwhile, the expressions in the denominators of equations (9) for the direct and inverse transformations have a direct connection in accordance with the Simultaneity Rule [2].

In this case (Y and Z coordinates are omitted to simplify the record), the Lorentz transformations in general form will have the following record:

$$x' = \frac{x - vt}{k_s} \qquad x = \frac{x' + vt'}{k'_s}$$

$$t' = \frac{t - \beta\frac{x}{v}}{k_s} \qquad t = \frac{t' + \beta\frac{x'}{v}}{k_s}$$

where: $k_s\, k'_s = 1 - \beta^2$
As we see, at ($\beta > 1$), when the condition ($k_s = |-k'_s|$) or ($|-k_s| = k'_s$), exists, the expression $[(1-\beta^2)^{1/2}]$ cannot be imaginary because it has its own solution:

$k_{s\,1,2} = \pm (|1-\beta^2|)^{1/2}$ at the same time: $k'_{s\,1,2} = (1-\beta^2)/\,k_{s\,1,2}$

It follows from this that the Lorentz transformations are performed even under the condition that ($\beta > 1$), i.e. at ($v > c$). In this case, the simultaneity coefficients (ks) and (k's) are applied only as opposite numbers: ($k_{s1}, -k'_{s1}$) or ($-k_{s2}, k'_{s2}$).

The described property of extracting the square root from a negative number in relation to the Lorentz transformation is important because it (the transformation) does not impose restrictions on the movement of any objects at a speed exceeding the speed of light. The SRT does not contain any other arguments for banning movement at superluminal speeds.

Conclusion

The result of extracting the square root from a negative number are real numbers

$(-d)^{1/2} = \pm a_{1,2}$

equal to the numbers extracted from the modulus of this negative number

$(|-d|)^{1/2} = \pm a_{1,2}$

in cases where the following condition is present:

$a_1\,(-a_2) = (-d\,)$ or $(-a_1)\,a_2 = (-d\,)$

that is, when the opposite values of the root are used simultaneously in the equation (system of equations): ($a_1, -a_2$), or ($-a_1, a_2$), the multiplication of which forms the same negative discriminant ($-d$).

References

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