ISSN: 3029-0724

Review Article

Journal of Environmental Science and Agricultural Research

Practical Problems with Function Extremes

Bianca-Elena Burlacu[1*], Andra-Silvia Dumbrava[1], Matei-Ioan Somesan[2] and Cosmin-Constantin Nitu Scientific Coordinator[1]

[1]*University of Agronomic Sciences and Veterinary Medicine of Bucharest, Faculty of Land Reclamation and Environmental Engineering, Bucharest, Romania*
[2]*National College "Mihai Viteazul", 62 Pache Protopopescu Blvd, District 2, 021407, Bucharest, Romania*

***Corresponding author**
Bianca-Elena Burlacu, University of Agronomic Sciences and Veterinary Medicine of Bucharest, Faculty of Land Reclamation and Environmental Engineering, Bucharest, Romania.

Received: July 16, 2025; **Accepted:** July 23, 2025; **Published:** August 01, 2025

ABSTRACT
Let $f: D \subset \mathbb{R}^n \to \mathbb{R}$ be a function and $a \in D$ a point. We say that a is a maximum (or a minimum) point for the function f if $f(x) \geq f(a)$ (or $f(x) \leq f(a)$ $\forall x \in D$. A maximum or a minimum point of a function is called an extreme point. In this paper we use the algorithms for determining the local extremes (conditional or unconditional) of a function to solve a variety of problems, mostly practical. These powerful methods are very useful and should be mastered by all students.

Keywords: Function Extreme Point, Maximum Point, Minimum Point, Practical Problems

Introduction

The study of extrema—maximum and minimum values—of functions plays a central role in mathematical analysis and its applications. Identifying these critical points is essential in understanding the behavior of functions and solving a wide range of real-world problems. From optimizing production costs and maximizing profits in economics, to minimizing energy use in engineering systems or determining the best trajectory in physics, the concept of function extrema provides powerful tools for modelling and decision-making.

This article explores the theoretical foundations of function extrema, including conditions for local and global maxima and minima, and examines their practical significance through concrete examples. Emphasis is placed on both unconstrained and constrained optimization, with applications that demonstrate how mathematical theory translates into effective solutions in diverse fields.

Materials and Methods

Definition 1. Let $f: D \subset \mathbb{R}^n \to \mathbb{R}$ be a function, $a \in A$ a point, and U a neighborhood of a. We say that a is a maximum local point for the function f if $f(x) \leq f(a)$, $\forall x \in U$. We say that a is a minimum local point for the function f if $f(x) \geq f(a)$, $\forall x \in U$.

Definition 2. Let $f: D \subset \mathbb{R}^n \to \mathbb{R}$ be a C^2 differentiable function (i.e. twice differentiable, with all $f''_{x_i x_j}$ continuous). The hessian matrix of the function f is

$$H_f = (f''_{x_i x_j})_{i,j=\overline{1,n}}$$

$$= \begin{pmatrix} f''_{x_1^2} & f''_{x_1 x_2} & f''_{x_1 x_3} & \cdots & f''_{x_1 x_n} \\ f''_{x_2 x_1} & f''_{x_2^2} & f''_{x_2 x_3} & \cdots & f''_{x_2 x_n} \\ f''_{x_3 x_1} & f''_{x_3 x_1} & f''_{x_3^2} & \cdots & f''_{x_3 x_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f''_{x_n x_1} & f''_{x_n x_1} & f''_{x_n x_1} & \cdots & f''_{x_n^2} \end{pmatrix}$$

Citation: Bianca-Elena Burlacu, Andra-Silvia Dumbrava, Matei-Ioan Somesan, Cosmin-Constantin Nitu Scientific Coordinator. Practical Problems with Function Extremes. J Envi Sci Agri Res. 2025. 3(4): 1-7. DOI: doi.org/10.61440/JESAR.2025.v3.73

We also consider the diagonal minors (sub determinants) of order i of H_f:

$$\Delta_i = \begin{vmatrix} f''_{x_1^2} & f''_{x_1x_2} & \cdots & f''_{x_1x_i} \\ f''_{x_2x_1} & f''_{x_2^2} & \cdots & f''_{x_2x_i} \\ \cdots & \cdots & \cdots & \cdots \\ f''_{x_ix_1} & f''_{x_ix_2} & \cdots & f''_{x_i^2} \end{vmatrix}$$

Theorem 1 (Unconditional Extremes): Let f: $D \subset \mathbb{R}^n \to \mathbb{R}$ be a C^2 differentiable function and $a = (a_1,a_2\ldots,a_n)$ a solution (called a stationary point or a critical point) of the system [1]

$$(s_1) : \{f'_{x_i} = 0, i = \overline{1,n}$$

Then

a) If $\Delta_i > 0$, $i = \overline{1,n}$, then a is minimum point for the function f;
b) If $(-1)^i\ \Delta_i > 0$, $i = \overline{1,n}$, then a is maximum point for the function f;
c) If $\Delta_2<0$, then a is not an extremum point for the function f and in we say that a is a saddle point

Remark 1. Consider the second order differential of the function f

$$d^2 f = \sum_{i=1}^{n} \frac{\partial^2 f}{\partial x_i^2} dx_i^2 + 2\sum_{i<j} \frac{\partial^2 f}{\partial x_i x_j} dx_i dx_j$$

As it can be seen in the proof of Theorem 1, an equivalent formulation for the minimum situation a) is $d^2f(a)>0$ (H_f is positively defined), and for the maximum situation b) $d^2f(a)<0$ (H_f is negatively defined).

Theorem 2 (conditional extremes) [2,3]
Let f: $D \subset \mathbb{R}^n \to \mathbb{R}$ be a C^2 differentiable function. We aim to find the extreme points of the function f satisfying the conditions

$$\varphi j = 0,\ j = \overline{1,m}$$

In this scope, we consider the associated *Lagrange function*

$$L(x,\lambda) = f(x) + \sum_{j=1}^{m} \lambda_j \varphi_j(x)$$

where $x=(x_1,x_2,\ldots,x_n) \in \mathbb{R}^m$ and $\lambda = (\lambda_1,\lambda_2,\ldots,\lambda_m) \in \mathbb{R}^m$.

Let $(a,\lambda)=(a_1,a_2,\ldots,a_n,\lambda_1,\lambda_2,\ldots,\lambda_m)$ be a solution of the system:

$$(S_1): \begin{cases} L'_{x_j} = 0, i = \overline{1,n} \\ L'_{\lambda_j} = 0, i = \overline{1,m} \end{cases}$$

$$\Leftrightarrow: \begin{cases} L'_{x_j} = 0, i = \overline{1,n} \\ L'_{\lambda_j} = 0, i = \overline{1,m} \end{cases}$$

In order to study the nature of the point $a=(a_1,a_2,\ldots, a_n)$, we differentiate the relations (*)

$$\sum_{i=1}^{n} \frac{\partial \varphi_j}{\partial x_i} dx_i = 0,\ j = \overline{1,m}$$

We write $dx_{n-p+1},dx_{n-p+2},\ldots dx_n$ in terms of $dx_1,dx_2,\ldots dx_{n-p}$, and replacing them in the relation $d^2L(a)= \sum_{k,l=1}^{n} L''_{x_kx_l}(a)dx_kdx_l$, we obtain the quadratic form

$$d^2L(a) = \sum_{k,l=1}^{n=p} L''_{x_kx_l}(a)dx_kdx_l$$

with the diagonal minors $\Delta_k, k = \overline{1,n-p}$.

Then

a) If $\Delta_k>0$, $i = \overline{1,n-p}$, then a is conditional minimum point for the function f;
b) If $(-1)^k\ \Delta_k>0$, $k = \overline{1,n-p}$, then a is conditional maximum point for the function f.

An important case of these theorems is the one for functions in a single variable:

Theorem 3. Let f: $I \subset \mathbb{R} \to \mathbb{R}$ be a derivable function, I an open interval, and $a \in I$ a solution (called a critical point or a stationary point) of the equation $f'(x)=0$.

Then

a) If there exists $\varepsilon>0$ such that $f'(x)<0$, $\forall\ x \in (a-\varepsilon, a)$, and $f'(x)>0$, $\forall\ x \in (a,a+\varepsilon)$, the a is a minimum local point of the function f.
b) If there exists $\varepsilon>0$ such that $'(x)<0$, $\forall\ x \in (a-\varepsilon, a)$, and $f'(x)>0$, $\forall\ x \in (a,a+\varepsilon)$, the a is a maximum local point of the function f.

Remark 2. The easiest way to study the extremes of function in one variable is with the help of a table called the variation table of the function

Table 1: General variation table

x	
$f(x)$	
$f'(x)$	

An improved version of Theorem 3, may be applied, when possible:

Theorem 4: Let f: $I \subset \mathbb{R} \to \mathbb{R}$ be C^2 class function, I is an open interval, and let a be a solution of the equation [4]

$f'(x)=0$. Then

a) If $f''(a)>0$, then a is a minimum local point of the function f;
b) If $f''(a)<0$, then a is a maximum local point of the function f.

Applications
Practical Example from Economics: Profit Maximization in a Manufacturing Firm

Suppose a firm produces a certain product, and the revenue function and cost function are as follows: The revenue function:

$V(x)=50x-0{,}5x^2$

(where x is the number of units produced and sold)

The cost function:
$C(x)=20x+100$

Find the maximal value of the profit.

Solution. We want to maximize the profit function, which is:

$P(x) = V(x)-C(x) = -0{,}5x^2+30x-100$

We compute its derivative

$P'(x)=-x+30$

We find the stationary points:

$P'(x) = 0 \Rightarrow -x+30 = 0 \Rightarrow x = 30.$

The variation table is

Table 2: Variation table for *P*(*x*)

x	0……30………………..∞
$P(x)$	
$P'(x)$	++++ 0 -- -- -- -- -- -- -

In conclusion, $x = 30$ is a local maximum point of the profit function.

Remark 3. An elementary solution can be given in this situation, considering the maximum point of the second degree function ax^2+bx+c, with $a<0$, which is given by the peak of the parbola $V\left(-\frac{b}{2a},-\frac{\Delta}{4a}\right)$.

Practical Example from Physics
Electricity: An RLC circuit in series [5]

We consider an RLC circuit in series, having a resistance $R=10\Omega$, an inductance $L=0{,}5H$, a capacitor $C=200\ \mu F$, a source of alternative sinusoidal tension $U(t)=100\ sin(100t)$

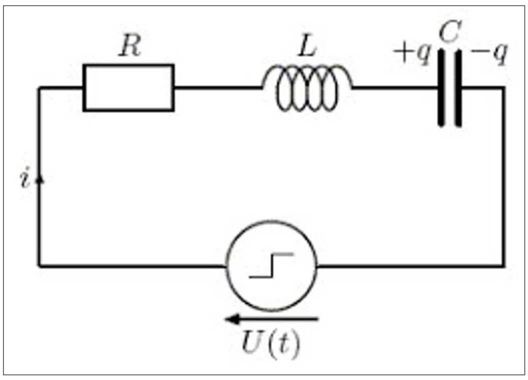

Figure 1: RLC circuit in series

The total impedance:
For ω=100, we have $X_L=\omega L=50\ \Omega$ and $X_C=\frac{1}{\omega C}=50\ \Omega$. Because $X_L-X_C=0$, it results that the circuit is in resonance, and the impedance is:

$$Z=\sqrt{R^2+(X_L-X_C)^2}=10\Omega$$

The maximum intensity is:

$$I_{max}=\frac{U_{max}}{Z}=10A$$

The intensity at the moment t is:

$i(t)=I_{max}\ sin(100t)=10\ sin(100t)$

We calculate its derivative:

$i'(t)=1000\ sin(100t)$

We compute the critical points:
$i'(t)=0 \Rightarrow cos(100t)=0 \Rightarrow 100t$

$$=\pm\frac{\pi}{2}+2k\pi=\frac{\pi}{2}+k\pi, k\in\mathbb{Z}$$

$$\Rightarrow t=\frac{1}{100}\left(\frac{\pi}{2}+k\pi\right), k\in\mathbb{Z}$$

Extreme values:
Because the derivative is a sinusoidal function, it's changing its sign and one has maximum values $i(t)= +10$ A and minimum values $i(t)= -10$ A.

Means inequality

$$\frac{x_1+x_2+\cdots x_n}{n}\geq\sqrt[n]{x_1,x_2,\cdots x_n},$$
$$\forall a_i\in\mathbb{R}_+,\ i=\overline{1,n}$$

The equality holds when $x_1=x_2=\ldots=x_n$.

Proof. Without loss of generality, we may assume that
$P=x_1x_2\ldots x_n=1$

We consider the function $f:\mathbb{R}^n\to\mathbb{R}$, setting the product as a constant

$$f(x_1,x_2,\cdots,x_n)=\frac{x_1+x_2+\cdots x_n}{n}$$

and we want to determine its extremes, conditioned by the relation
$\varphi(x)=x_1x_2\ldots x_n-1=0$

This is a conditional extreme problem. We consider the Lagrangian function
$L(x,\lambda)=f(x)+\lambda(x_1x_2\ldots x_n-1)$

For $x_i=0$, $i=\overline{1,n}$ the equality is verified. Let's assume that $P\neq 0$. We consider the system:

$$(S_2): \begin{cases} L'_{x_j} = 0, i = \overline{1,n} \\ L'_\lambda = 0 \end{cases}$$

$$\Leftrightarrow: \begin{cases} \frac{1}{n} + \frac{\lambda}{x_i} = 0, i = \overline{1,n} \\ x_1 x_2 \ldots x_n = 1 \end{cases}$$

From here, we deduce that

$x_1 = x_2 = \cdots = x_n = 1$ and $\lambda = -\frac{1}{n}$

Let

$$L(x) = \frac{x_1 + x_2 + \cdots x_n}{n} - \frac{1}{n} x_1 x_2 \cdots x_n + 1$$

Its derivatives are:

$$L''_{x_k x_l}(1,1\ldots1) = -\frac{1}{n}, \ if\ k \neq l$$

$$L''_{x_k^2} = 0$$

$$d^2L(1,1\ldots1) = \sum_{k,l=1}^{n} L''_{x_k x_l}(1,1,\ldots.1) dx_k dx_l$$

$$= -\frac{1}{n} \sum_{k \neq l}^{n} dx_k dx_l$$

By differentiating the condition at (1,1,..,1):

$$\sum_{k=l}^{n} dx_k = 0 \Rightarrow dx_k{}^2 = \sum_{k \neq l}^{n} dx_k dx_l$$

$$d^2L(1,1\ldots1) = \frac{1}{n} \sum_{k \neq l}^{n} dx_k{}^2 > 0$$

In conclusion (1,1,..,1) is a minimum conditional point.

a) From all the Rectangles of Constant Perimeter, Determine the One of Maximum Area
b) From all the Rectangles of Constant Area, Determine the One pf Minimum Perimeter

Solution [6]
a) Be denoting the sides of the rectangle with $L=x>0$ and $l=y>0$, and the constant with $2C$, we have to solve the conditional extreme problem

$\max A = \max f(x,y) = xy$
$x+y = C$

But it is a lot easier in this situation to solve this problem elementary via the means inequality:

$$\frac{x+y}{2} \geq \sqrt{xy}$$

which implies

$$\frac{C}{2} \geq \sqrt{xy} \Rightarrow xy \leq \frac{C^2}{4}$$

The equality holds for $x = y = \frac{C}{2}$ when the rectangle is a square.

b) Be denoting the sides of the rectangle with $L = x>0$ and $l = y>0$, and the constant with P, we have to solve the conditional extreme problem

$\max A = \max f(x,y) = x+y$
$xy = P$

But it is a lot easier in this situation to solve this problem elementary via the means inequality:

$$\frac{x+y}{2} \geq \sqrt{xy}$$

which implies

$$\frac{x+y}{2} \geq \sqrt{P}$$

The equality holds for $x = y = \sqrt{P}$, when the rectangle is a square.

Determine the dimensions of a parallelepipedal box (without lid), with the volume equal to a^3 such that the surface of metal sheet is minimal.

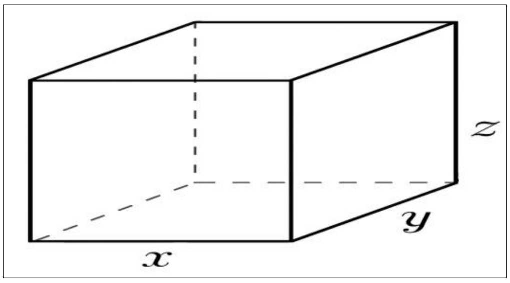

Figure 2: Parallelipiped

Solution [7,8]
The geometric problem is

$\min A = Ll+2Lh+2lh$
$V = Ll00h=a^3$

We denote by x,y and z the sides of the parallelipiped
$\min A = \min f(x,y,z) = xy+2xz+2yz$
$V = xyz = a^3$

where $x,y,z>0$
The condition is
$\varphi(x,y,z) = xyz-a^3=0$.

The Lagrange function is
$L(x,y,z,\lambda) = f(x,y,z)+ \lambda\varphi(x,y,z)$
$L(x,y,z,\lambda) = xy+2xz+2yz+\lambda(xyz-a^3)$

We consider the system:

$$(S): \begin{cases} L'_x = 0 \\ L'_y = 0 \\ L'_z = 0 \\ L'_\lambda = 0 \end{cases} \Leftrightarrow \begin{cases} y + 2z + \lambda yz = 0 \\ x + 2z + \lambda xz = 0 \\ 2x + 2y + \lambda xy = 0 \\ xyz - a^3 = 0 \end{cases}$$

If $x \neq y$ then by substracting the first two equations, we obtain:

$y-x=-\lambda z(y-x)|:y-x \Rightarrow 1=-\lambda z$

$\Rightarrow z=-\frac{1}{\lambda}$

In the first equation, replacing $\Rightarrow z=-\frac{1}{\lambda}$, we get z = 0, which is false.

In conclusion, we must have $x = y$, so the system (S) becomes:

$$\begin{cases} x+2z+\lambda xz=0 \\ 4x+\lambda x^2=0 \Rightarrow x=-\frac{4}{\lambda} \\ x^2 z=a^3 \Rightarrow z=\frac{a^3\lambda^2}{16} \end{cases}$$

Replacing x and z in the first equation, we find the Lagrange multiplier

$$\lambda=-\frac{2\sqrt[3]{4}}{a}$$

and the solutions:

$$x=y=\frac{2a}{\sqrt[3]{4}}, \quad z=\frac{a}{\sqrt[3]{4}},$$

So, the stationary point is

$$P=\left(\frac{2a}{\sqrt[3]{4}},\frac{2a}{\sqrt[3]{4}},\frac{a}{\sqrt[3]{4}}\right)$$

The Lagrange function is

$$L(x,y,z)=xy+2xz+2yz-\frac{2\sqrt[3]{4}}{a}(xyz-a^3)$$

We have $L''_{x^2}=L''_{y^2}=L''_{z^2}=0$, and

$$L''_{xy}=1-\frac{2\sqrt[3]{4}}{a}z, \quad L''_{xz}=2-\frac{2\sqrt[3]{4}}{a}y, \quad L''_{yz}=2-\frac{2\sqrt[3]{4}}{a}x$$

and the differential of second order is

$$d^2L(P)=2L''_{xy}(P)dxdy+2L''_{xz}(P)dxdz+2L''_{yz}(P)dydz=-2dxdy-4dxdz-4dydz$$

By differentiating the condition at the point P, we have:

$$dz=-\frac{1}{2}dx-\frac{1}{2}dy$$

Thus, $d^2L(P)=2dx^2+2dy^2+2dxdy>0$, so $d^2L(P)$ is positively defined, which shows that P is *a* minimum point.

Determine the dimensions of circular cone of maximum volum if it is inscribed in a sphere of radius R.

Solution. The geometrical problem is

$$maxV=\frac{\pi r^2 h}{3}$$

$(h-R)^2+r^2=R^2$

We denote by $h=x+R$. The problem becomes:

$$maxV=f(x,r)=\frac{\pi r^2(x+R)}{3}$$

conditioned by

$x^2+r^2=R^2$

where $x,r>0$.

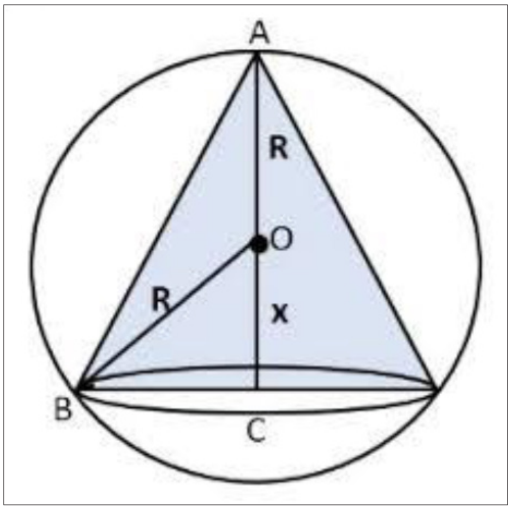

Figure 3: Cone inscribed in a sphere

The condition is $\varphi(x,r)=x^2+r^2-R^2$

Lagrange's function is $L(x,r,\lambda)=f(x,r)+\lambda\varphi(x,r)$

$$L(x,r,\lambda)=\frac{\pi r^2(x+R)}{3}+\lambda(x^2+r^2-R^2)$$

We consider the system

$$(S): \begin{cases} L^1_x=0 \\ L^1_r=0 \\ L^1_\lambda=0 \end{cases} \Leftrightarrow \begin{cases} \frac{\pi r_2}{3}+2\lambda x=0 \\ \frac{2\pi(x+R)}{3}+2\lambda r=0 \\ x^2+r^2-R^2=0 \end{cases}$$

The second equation can be written:

$$2r\left[\frac{\pi(x+R)}{3}+\lambda\right]=0|:r\neq 0$$

$$\Rightarrow x=-\frac{3\lambda}{\pi}-R$$

From the first equation, we obtain:

$$r^2=\frac{-6\lambda x}{\pi}=\frac{18\lambda^2}{\pi}+\frac{6\lambda R}{\pi}$$

Replacing into the third equation, we obtain the second degree equation

$9\lambda^2+4\pi R\lambda=0$

The Lagrange multiplier is the positive solution

$$\lambda=-\frac{4\pi R}{9}$$

Thus, $x=\frac{R}{3}, r=\frac{2\sqrt{2}R}{3}$ and the stationary point is

So, the stationary point is

$$P=(x,r)=\left(\frac{R}{3},\frac{2\sqrt{2}R}{3}\right)$$

Lagrange's function is

$$L(x,r)=\frac{\pi r^2(x+R)}{3}-\frac{4\pi R}{9}(x^2+r^2-R^2)$$

We have $L''_{x^2}=-\frac{8\pi R}{9}$, $L''_{r^2}=\frac{2\pi(x+R)}{9}-\frac{4\pi R}{9}=0$, and $L''_{xr}=\frac{2\pi R}{3}$

The differential of second order is

$$d^2L(P) = L''_{x^2}(P)dx^2 + 2L''_{xz}(P)dxdr + L''_{r^2}(P)dr^2$$

By differentiating the condition, we have:

$$\varphi'_x dx + \varphi'_r dr = 0 \Rightarrow dr = -\frac{x}{r}dx$$

From here $d^2L(P) = -\frac{4\pi R}{3}dx^2 < 0$, so $d^2L(P)$ is negatively defined, which shows that P is a maximum point.

Determine the dimensions of cilinder of maximum total area if it is inscribed in a sphere of radius R.

Solution. The geometric problem is

$\max A_t = 2\pi rh + 2\pi r^2$

$h^2 + (2r)^2 = (2R)^2$

We denote $h=2x$ and we have to determine

$\max A_t = f(x,r) = 2\pi r(2x+r)$

conditioned by

$x^2 + r^2 = R^2$

where $x,r>0$.

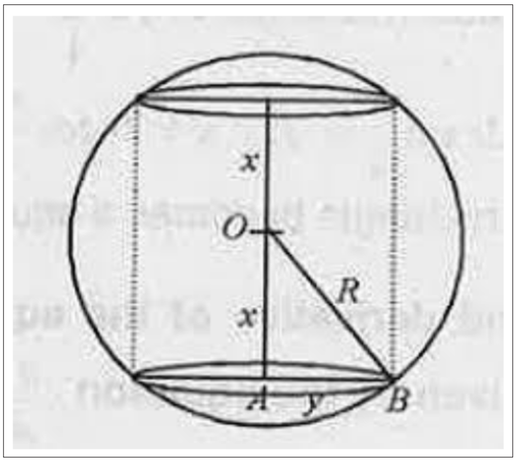

Figure 4: Rectangular cilinder inscribed in a sphere

The condition is

$\varphi(x,r) = x^2 + r^2 - R^2$

Lagrange's function is

$L(x,r,\lambda) = f(x,r) + \lambda\varphi(x,r)$

$L(x,r,\lambda) = 2\pi r(2x+r) + \lambda(x^2 + r^2 - R^2)$

We consider the system:

$$S: \begin{cases} L'_x = 0 \\ L'_r = 0 \\ L'_\lambda = 0 \end{cases} \Leftrightarrow \begin{cases} 4\pi r + 2\lambda x = 0 \\ 4\pi x + 4\pi r + 2\lambda r = 0 \\ x^2 + r^2 - R^2 = 0 \end{cases}$$

From the first equation $x = -\frac{2\pi r}{\lambda} > 0$ and replacing it into the second equation we obtain the second order equation

$\lambda^2 + 2\pi r - 4\pi^2 = 0$

We find

$\lambda_{1,2} = -(1 \pm \sqrt{5})\pi$

Because $\lambda<0$, we have

$\lambda = -(1+\sqrt{5})\pi$

Thus,

$$x = \frac{2r}{(1+\sqrt{5})}$$

Replacing x in the third equation, we get

$$r = \frac{R\sqrt{1+\sqrt{5}}}{\sqrt{2\sqrt{5}}}, \quad x = \frac{2R}{\sqrt{10+2\sqrt{5}}}$$

Therefore, the stationary point is

$$P = (x,r) = \left(\frac{2R}{\sqrt{10+2\sqrt{5}}}, \frac{R\sqrt{1+\sqrt{5}}}{\sqrt{2\sqrt{5}}}\right)$$

The Lagrange function is

$L(x,r) = 2\pi r(2x+r) - (1+\sqrt{5})\pi(x^2 + r^2 - R^2)$

We have $L''_{x^2} = 2\lambda$, $L''_{r^2} = 4\pi r + 2\lambda - \frac{4\pi r}{p} = 0$ and $L''_{xr} = 4\pi$ the differential of second order is

$$d^2L(P) = L''_{x^2}(P)dx^2 + 2L''_{xz}(P)dxdr + L''_{r^2}(P)dr^2$$

By differentiating the condition, we have: $dr = -\frac{x}{r}dx$

Therefore,

$$d^2L(P) = -\left(\left(2+2\sqrt{5}\right)\pi + \frac{16}{1+\sqrt{5}}\pi^2\right)dx^2 < 0,$$

so $d^2L(P)$ is negatively defined, which shows that P is a maximum point.

Conclusions

The study of function extrema is a fundamental component of mathematical analysis, with broad applications in both theoretical and applied contexts. Identifying local and global maxima and minima enables optimization of processes, modeling of natural or economic phenomena, and deeper understanding of critical system behavior. Analytical tools such as derivatives and extremum conditions, complemented by numerical and geometric techniques, provide a robust framework for such investigations. The wide applicability of these concepts in fields like physics, economics, engineering, and computer science highlights the interdisciplinary nature of extremum analysis. Ultimately, mastering the methods for determining function extrema is a key competency for researchers and professionals working with mathematical models.

References

1. Boboc N. Mathematical Analysis, University of Bucharest Publishing House. 1999.
2. Colojoară I. Mathematical Analysis. E.D.P., Bucharest. 1983.
3. Nicolescu M, Dinculeanu N, Marcus S. Mathematical Analysis, University of Bucharest Publishing House. 1971.
4. Andronache M, Șerbănescu D, Țena M. Mathematics Textbook for the 11th grade M1, Ed. Art, Bucharest. 2006.
5. Matea C, Garabet M. Physics. Textbook for the 11th grade F1, Ed. All, Bucharest. 2012.
6. Burlacu R. Mathematical Analysis, University of Bucharest Publishing House. 2006.

7. Martin O. Problems of Mathematical Analysis, Ed. Matrix Rom, Bucharest. 2008.
8. Nițu CC. Differentials and applications to function approximations, Scientific Papers. Series E. Land Reclamation, Earth Observation & Surveying. 2023.