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Review Article

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General Rule That Oblique Projectile Motion Has the Longest Range Generalization and Application of a Classic Exercise

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Annotation
The range of an object in oblique projectile motion is often a question that students are more interested in. During the teaching process, the author found that no matter it is thrown along a horizontal plane or thrown upward (or downward) along an inclined plane, the projectile angle and the maximum range of the object when the object has the longest range have a relatively simple unified law.

Keywords: Oblique Projectile Motion; Maximum Range; General Rules

Classic Original Question
An object is thrown from a horizontal plane with a constant speed v. Ignoring air resistance, try to find the maximum range of the projectile and the corresponding angle of throw [1].

Solution: As shown in Figure 1, in the projectile plane (vertical plane), take the horizontal direction and vertical direction as the x-axis and y- axis directions respectively, and establish the rectangular coordinate system Oxy. Then the component equations of the projectile motion are:

$$v_x = v\cos\theta \quad v_y = v\sin\theta - gt \tag{1}$$

$$x = \left(v\cos\theta\right)t - \frac{1}{2}gr^2 \tag{2}$$

To find the range of the projectile, we can take the value of x when y=0 from equation (2) and obtain:

$$L = \frac{v^2 \sin 2\theta}{g} \tag{3}$$

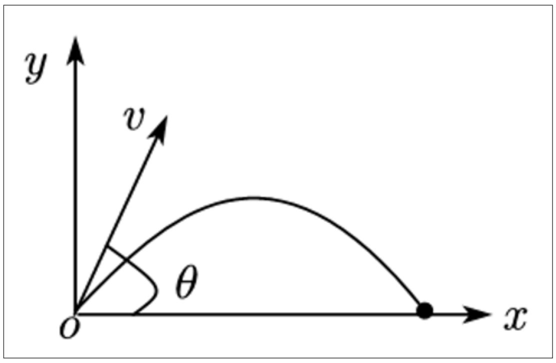

Figure 1

From the expression of L, it is easy to get the maximum range and the corresponding projection angle:

$$L_m = \frac{v^2}{g} \theta_m = \frac{\pi}{4} \tag{4}$$

That is, when the range is the longest, the velocity direction of the projectile angle should be on the bisector of the angle between the vertical direction and the horizontal plane. In fact, this rule still applies to throwing objects upward (or downward) along an inclined plane, but it should be modified to when the range is the longest, the velocity direction of the projectile angle should be

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on the bisector of the angle between the vertical direction and the inclined plane. Now we will prove it for general cases.

Conclusion and Extension

Now let's discuss the range of a projectile moving upward (or downward) along an inclined plane. As shown in Figures 2 and 3, let the x and y axes of the rectangular coordinates point upward (or downward) along the inclined plane and perpendicular to the inclined plane, respectively. Decompose the velocity and gravitational acceleration. At this time, x, y the motions in the x and y directions are all uniformly accelerated linear motions, and their motion equations in the x and y directions are [2]:

$$v_x = v\cos\theta \pm \left(g\sin\phi\right)t \quad (5)$$

$$v_y = v\sin\theta - \left(g\cos\phi\right)t \quad (6)$$

$$x = \left(v\cos\theta\right)t \pm \frac{1}{2}\left(g\sin\phi\right)t^2 \quad (7)$$

$$y = \left(v\sin\theta\right)t - \frac{1}{2}\left(g\cos\phi\right)t^2 \quad (8)$$

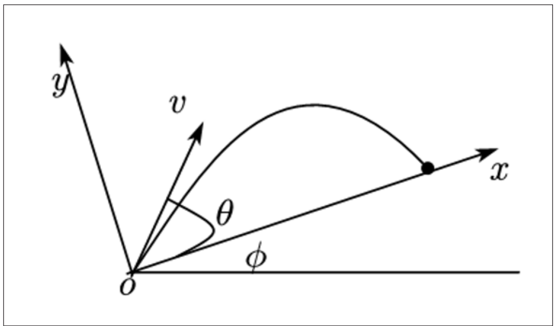

Figure 2

equations (5) and (7), the positive sign indicates that the throw is made downward along the inclined plane, and the negative sign indicates that the throw is made upward along the inclined plane. In Figure 2, to find the range of the projectile along the slope, we can take the x value when y=0 from equation (8) and obtain:

$$L = \frac{2v^2\cos\left(\theta+\phi\right)\sin\theta}{g\cos^2\phi} \quad (9)$$

Perform product and difference transformation on the factors $\cos\left(\theta+\phi\right)\sin\theta$ contained in formula (9): θ

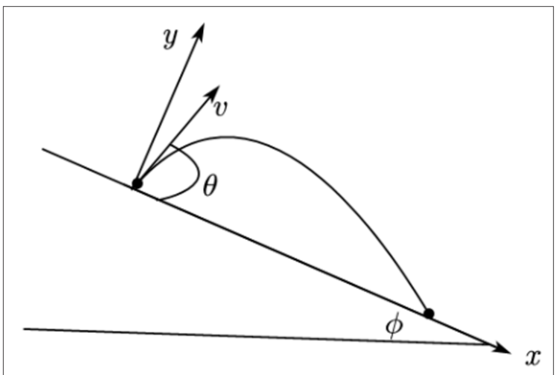

Figure 3

$\cos\left(\theta+\phi\right)\sin\theta = \frac{\sin(2\theta+\phi)-\sin\phi}{2}$, Yizhi, at that time $2\theta+\phi=\frac{\pi}{2}$

L takes the maximum value:

$$L_m = \frac{v^2}{g}\cdot\frac{1-\sin\phi}{cas^2\phi} = \frac{v^2}{g\left(1+\sin\phi\right)} \quad (10)$$

The corresponding projection angle is: $\theta_m = \frac{1}{2}\left(\frac{\pi}{2}-\phi\right)$ (11)

That is, when the range is the longest when shooting upward along the slope, the velocity direction of the projectile angle should be on the vertical direction and the angle bisector of the slope.

Similarly, to find the range of the projectile along the slope in Figure 3, we can take the x value when y=0 from equation (8) and get: $L = \frac{2v^2\cos\left(\theta-\phi\right)\sin\theta}{g\cos^2\phi}$, (12), and also perform product-to-sum and difference processing to get:

$$L_m = \frac{v^2}{g}\cdot\frac{1+\sin\phi}{\cos^2\phi} = \frac{v^2}{g\left(1-\sin\phi\right)} \quad (13)$$

The corresponding projection angle is: $\theta_m = \frac{1}{2}\left(\frac{\pi}{2}+\phi\right)$ (14)

That is, when the distance of the shot is the longest along the slope, the velocity direction of the projectile angle should be on the vertical direction and the angle bisector of the slope.

In summary, the maximum range of a projectile along an inclined plane (including a plane) is: $L_m = \frac{v^2}{g\left(1\pm\sin\phi\right)}$, and the corresponding projectile angle is on the bisector of the angle between the vertical direction and the inclined plane (including the plane): $\theta_m = \frac{1}{2}\left(\frac{\pi}{2}\mp\phi\right)$, where for the plane, $\phi=0, L_m=\frac{v^2}{g}, \theta_m=\frac{\pi}{4}$ is the situation in the classic example.

Specific Application

(9th Competition) $\alpha = 30^o$ A ski jumping competition is held on an inclined snow slope, as shown in Figure 4. The athlete starts to slide down from point A on the slope. When he reaches the take-off point O, he uses equipment and skills to maintain the same speed at that point and θ jumps at an angle to the horizontal, and finally lands at point B on the slope. The distance L between points OB on the slope is the record of this sport. It is known that point A is h=50m higher than point O. Ignoring various resistances and frictions, how far can the athlete jump? What is the take-off angle at this time [3]?

Solution: The speed of the athlete at point O.

$v_0 = \sqrt{2gh} = 10\sqrt{10}m/s$ After taking off, the athlete performs an oblique projectile motion. According to the general rule that oblique projectile motion has the longest range, we can get: $L_m = \frac{v_0^2}{g\left(1-\sin\alpha\right)} = 200m$, the take-off angle in Figure 4 is not the projectile angle, but $\theta+\alpha$ it is the projectile angle mentioned in the previous article. According to the angle bisector law, it is easy to get $\theta=30^o$

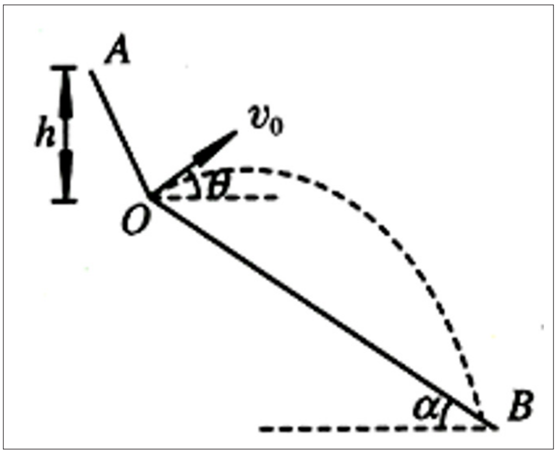

Figure 4

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