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Analytical Modeling Enables Quantifying the Probabilities of the Outcome of Critical Complex Maritime Missions and Systems: Perspective and Examples

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"There is nothing more practical than a good theory."

Kurt Lewin (1890-1947), German-born American Social Psychologist

"The practical value of mathematics is, in effect, a possibility to obtain, with its help, results simpler and faster."

Andrey N. Kolmogorov (1903-1987), Russian Mathematician

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ABSTRACT

"Mathematical" and, particularly, probabilistic modeling enables shedding useful light on some critical complex safety-at-sea and off-shore tasks and problems. The approach does that by predicting the most likely outcomes of planned missions including those of the "human-in-the-loop" type. In such missions the reliability of the instrumentation and equipment, both its hard- and software, and human performance contribute jointly to the mission's outcome. Two maritime-safety problems, in which the role of the human factor is taken into account, are addressed in this write-up as suitable examples: 1) predicted probability of failure of a planned maritime mission/voyage and 2) probabilistic assessment of the possible roles of the more or less permanent human factors (such as age, experience, education, health, training, etc.) and the temporary state of the human's health (say, such as, e.g., cold, headache, runny nose) and/or mind (say, short-term loss of attention, drowsiness) that might affect the likelihood of the occurrence of what is known as human error. It is concluded that while some kind of predictive modeling should always be considered and conducted prior to and, whenever possible and appropriate, also during accelerated reliability testing of the navigation instrumentation or human performance, physically meaningful analytical ("mathematical"), preferably probabilistic predictive models should be developed and applied to complement the results of computer simulations: these two major modeling tools are based on different assumptions, employ different calculation techniques, and if the results obtained using these tools agree, then there is a good reason to believe that the obtained information is sufficiently accurate and, hence, trustworthy. Future work should address other suitable applications of the employed analytical modeling technique, as well as the development of practical analytical models for establishing the risks for critical complex systems and applications, considering both the predicted never zero probabilities and the consequences of the possible failures. These probabilities cannot be high, of course, but they should not be lower than necessary either: they should be adequate for the system and application. Systems that "never fail" are much more expensive than they could and should be.

Keywords: Boltzmann-Arrhenius-Zhurkov (BAZ) equation, Double-exponential-probability-distribution-function (DEPDF), Failure-oriented-accelerated-testing (FOAT), Figures-of-merit (FoM), Finite-element-analyses (FEA), Highly-accelerated-life-testing (HALT), Human-capacity-factor (HCF), Human-computer interactions (HCI), Human error (HE), Human-in-the-loop (HITL), Human non-failure (HnF), Human-system-interaction/integration (HSI), Mean-time-to-error (MTTE), Mean-time-to-failure (MTTF), Mental (cognitive) workload (MWL), Probabilistic-design-for-reliability (PDfR) concept, Probabilistic predictive modeling (PPM), State-of-health (SoH)

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INTRODUCTION

Analytical ("Mathematical") Modeling: Role, Significance, Attributes, Merits, Challenges

The role, significance, attributes, merits and challenges of analytical ("mathematical") and, particularly, probabilistic modeling employed in this analysis have been recently addressed and discussed in detail in application to various complex, mostly aerospace related, electronic and photonic systems, including various "human-in-the-loop" (HITL) and even astrobiology, medical-and-clinical, spacecraft-safety and the likelihood of Tunguska-type of event related problems and situations [1-28]. Analytical modeling occupies a special place in reliability and ergonomics engineering, because it enables obtaining simple relationships that explain the physics of failure and, particularly, paradoxical, i.e., a-priori non-obvious, phenomena better than computer simulations or even experimentation can. As Heinrich Hertz (1857-1894), the famous German physicist, had indicated, "mathematical formulas have their own life, they are smarter than we, even smarter than their authors, and often provide more than what has been expected from them." We live in the era of computers and apply computer simulations to model all the problems of importance we deal with. Using analytical modeling helps avoiding an undesirable Abraham Maslow's situation that "if the only tool you have is a hammer, you tend to see every problem as a nail." Computer-aided finite-element-analyses (FEA), e.g., initially implemented in the mid-1950s of the last century in the areas of engineering, where structures of complicated geometry were employed, such as, e.g., maritime, aircraft, and some civil engineering structures, have become shortly, owing to the progress in computer science, the major modeling tool in other areas of engineering as well, including physical design of electronic and photonic systems. Powerful and flexible FEA computer programs enable us to obtain, within a reasonable time, a solution to almost any stress-strain-related problem, but, as has been indicated, additional and independent modeling is highly advisable to make sure that it is also *the* solution. The computer-aided and the "old-fashioned" analytical ("mathematical") modeling tools are based on different assumptions, use different calculation techniques, and if the results obtained employing these tools agree, then there is a good reason to believe that the obtained information is accurate and trustworthy.

A paramount requirement for an effective and practical analytical model is simplicity, clear physical meaning, and, as Einstein had put it, "external justification and internal perfection". A good analytical model should be confirmed, of course, experimentally and should be based on simple relationships, clearly indicating the role of the major factors affecting the critical undertaking, mission, technology, object or structure. One authority in applied physics remarked, perhaps only partly in jest, that "the degree of understanding a physical phenomenon is inversely proportional to the number of variables used for its description", and the Ukrainian philosopher Gregory Skovoroda (1722-1794) had asserted, also only partly in jest, of course, that "we must be grateful to God for creating the world in such a way that everything simple is true and everything complicated is untrue." These statements are exaggerations, of course, but Hooke's law $\sigma = E\varepsilon$ (1678) in the strength-of-materials, Newton's laws in dynamics ("natural philosophy") and $F = G \frac{m_1 m_2}{r^2}$

(1687) in universal gravitation, Ohm's law $V=IR$ (1827) in electrical engineering, Lorentz' factor $\gamma = \left(\sqrt{1 - \frac{v^2}{c^2}} \right)^{-1}$ (1892) in astrophysics and special relativity, and Einstein's formula $E=mc^2$ (1905) in the general relativity theory are, probably, the best illustrations to these statements. Empirical relationships, such as, e.g., the numerous Coffin-Manson's type ones, widely used in electronics and photonics reliability engineering to assess the useful lifetime of solder-joint interconnections, the bottleneck of microelectronics and photonics structural reliability, are certainly useful, but their structure and particularly the non-integer exponents in the experimentally obtained and suggested expressions clearly indicate on the lack of understanding of the underlying physics of the solder material failure [29]. In this connection I would like to indicate a simple formula (see for details) [3] that enables determining the length of the inelastic zone, if any, at the end of a soldered interface in an electronic assembly: $l_y = \frac{1}{k} \left(\frac{\tau_{\max}}{\tau_y} - 1 \right)$ Here τ_y is the solder material's yield stress in shear, $\tau_{\max} = k \frac{\Delta\alpha\Delta T}{\lambda}$ is the maximum thermally induced elastic shearing stress (obtained assuming that no inelastic deformations take place), $\Delta\alpha\Delta T$ is the thermal strain (the product of the difference $\Delta\alpha$ in the CTEs of the soldered components and the change ΔT in temperature from the soldering temperature, when the induced stress is next-to-zero, and the low temperature of importance), λ is the longitudinal compliance of the assembly (the solder material does not contribute to it), $k = \sqrt{\frac{\lambda}{\kappa}}$ is the parameter of the shearing stress, and κ is the assembly's interfacial compliance in shear (both the bonded components and the solder layer play a role in it). If the above formula for the l_y results in a negative value, this simply means that no inelastic strains occur. The analytical predictions are in excellent agreement with the experiment and FEA data [30].

Human Factor in a Complex System of Maritime Safety

The critical role of the human factor in aerospace and maritime safety tasks and problems have been recently addressed [10-15,18,19,21,31-36], following Chauvin's publication [34]. As Wikipedia indicates, "complex systems is often used" as a broad term encompassing a research approach to problems in many diverse disciplines, such as transportation or communication systems, complex software and electronic systems, social and economic organizations, global climate, various organisms, the human brain, traffic, financial markets, opinion formation and spreading, internet and social media, transportation or communication systems, complex software and electronic systems, social and economic organizations (like cities), an ecosystem, a living cell, and, ultimately, the entire universe. Complex systems are systems whose behavior is intrinsically difficult to model a broad interdisciplinary field that examines how the interaction of many parts of the system and external interference can give rise to its holistic collective behavior. This popular and extraordinarily broad field includes practically all areas of physics, engineering, ergonomics, and even biology and cosmology. Because such systems appear in a wide variety of fields, the commonalities among them have become the topic of an independent area of research. Complex systems modeling is defined by the application of diverse mathematical, statistical and

computational techniques to generate insight into how some of the most complicated physical and natural systems in the world function. Complex system analyses are seldom straightforward and use diverse analytical, statistical and computational techniques to generate insight into how these systems actually function and perform.

There is a flood of literature on complex systems (see, e.g., [37-39], just to mention a few). Complex system theories have their roots in the numerous "theories of chaos" (see, e.g., [40-43]), which, in turn, have originated from the great French mathematician Henri Poincaré's work on "three-body problem and the equations of dynamics" [44] (see also [45]). It is important to emphasize that in these and in a number of other publications, including the present one, the word "chaos" is viewed as an extremely complicated and difficult-to-analyze-and-to-quantify problem, rather than as an absence of order. Note that some maritime safety problems were addressed by the first author of this write-up in the past as complex random systems [1, 46-50].

PERSPECTIVE

Probabilistic Design for Reliability (PDfR) Concept in Microelectronics and Photonics Engineering

The probabilistic-design-for-reliability (PDfR) concept [1,6,8,9,15,20] was initially developed in application to various electronics and photonics reliability physics tasks and problems and then applied to the human-in-the-loop (HITL), mostly in aerospace [10,18,19,21], and other vehicular missions and extraordinary situations and problems. The concept has its experimental foundation in the failure-oriented-accelerated-testing (FOAT) techniques [22, 23]. While FOAT at the manufacturing stage, known as burn-in-testing (BIT), is always conducted, even for commercial electronics and photonics products, and often also at the product development stage (shear-off tests and temperature cycling tests are good examples), the design stage FOAT is supposed to be conducted when a new technology, or a new design, or a new application of an existing technology or a design is considered and when no acceptable highly-accelerated-life-testing (HALT) procedures exist yet, nor suitable "best practices" have been established and agreed upon, and when there is a need and an intent to evaluate the useful lifetime of a product and the corresponding probability of its field failure [23]. This probability is, in effect, never zero, but, using the PDfR concept and the design-stage FOAT, could be made low enough to be adequate for the given product, system and application. The recently suggested multi-parametric Boltzmann-Arrhenius-Zhurkov (BAZ) equation [8, 9, 16, 20] can be used to predict the probability of failure from the design-stage FOAT data. It is noteworthy that this type of FOAT has been suggested and considered in addition to the manufacturing-stage-FOAT, known as "burn-in-testing (BIT)" (see, e.g., [7, 22, 23]) that is routinely conducted for any electronic, photonic or MEMS product, and also in addition to the development-stage-FOATs, such as, e.g., shear-off testing or temperature cycling, which are widely used in microelectronics engineering. The development stage FOAT is conducted to make sure that the considered technological and design approach and materials selection are acceptable, while the BIT type FOAT is conducted to get rid of "freaks", low reliability products, prior to shipping

the healthy ones, i.e., those that survived BITs, to the customer(s). It has been shown [9] that the multi-parametric BAZ model can be applied not only to the electronic packages and systems, but also to electronic devices, where the reliability of the p-n junction is critical. The multi-parametric BAZ equation (see next section for details) was initially suggested for the prediction of the lifetime of IC packages and devices [8, 9] and then applied, as a suitable analogy, in electronic manufacturing [7], ergonomics[10-15], space biology [16], medical-and-clinical [17, 25, 26] problems with an objective to establish the required level of the human capacity factor (HCF) [5,10-15] in various ergonomics-engineering human-in-the-loop undertakings. In this analogy, the activation energy (the "strength") in the BAZ equation plays the role of the HCF (the human's "bearing capacity") in the ergonomics-engineering formulation, and the thermal energy, defined in the BAZ equation as the product of the Boltzmann's constant and the absolute temperature, reflects the role and the level of the mental (cognitive) workload (MWL) [13,18,19,21]. Challenges that an aircraft pilot faces in an extraordinary situation are analogous to those that a surgeon copes with during a surgical operation [25]. This is true also in an extraordinary maritime navigation situation. Analogies associated with the role of the human factor and the state of his/hers health were addressed in [27,28]: simple experiments based on the probabilistic interpretation of the deterministic Fitts' law in the theory of human-computer interactions (HCI) could be conducted, considering their analogy with some critical outer space phenomena, such as the likelihood of a spacecraft collision with an asteroid [27] and the probability of the Tunguska-meteorite type of an event [28].

Multi-Parametric Boltzmann-Arrhenius-Zhurkov (BAZ) Equation in Human-System Interaction Problems

A physically meaningful multi-parametric Boltzmann-Arrhenius-Zhurkov (BAZ) equation [8, 9,16]

$$P = \exp \left[-\gamma_c C t \exp \left(-\frac{1}{k_B T} \left(U_0 - \sum_{i=1}^n \gamma_i \sigma_i \right) \right) \right] \quad (1)$$

for the probability of non-failure, could be employed to interpret and to quantify the test data of electronic and photonic electronic and photonic devices and systems, including, of course, those employed in maritime instrumentation and equipment. In this equation γ_c is the i -th stressor, γ_i is its sensitivity factor, U_0 is the activation energy, T is (absolute) temperature, k_B is Boltzmann's constant, C is the continuously monitored and measured (during the FOAT procedure) response (such as, e.g., leakage current or electrical resistance or any other suitable and measurable reliability feedback) of the system and σ_i is the sensitivity factor for this response. The expression in the inner parenthesis is the actual, effective, activation energy (the term has been coined by Arrhenius), and the product in the nominator is thermal energy. Thus, the expression (1) reflects the effect of the ratio of the effective activation energy (that reflects the "bearing capacity" of the given complex system) to the thermal energy (that reflects the external loading on the system). The response provides information about the degree of degradation (current damage level) during the FOAT, and the remaining "distance" (time, damage) from its a-priori decided-and-agreed-upon level viewed as an adequate indication of failure. The model (1) can be obtained by combining

Boltzmann's distribution in classical thermodynamics, Arrhenius equation in physical chemistry and Zhurkov's extension of the Arrhenius equation in experimental fracture mechanics [8]. The appropriate stressors in (1) could be any stimulus that shorten the useful lifetime of a device, package, module or system.

Non-Thermal Look at the BAZ Equation: Double-Exponential-Probability-Distribution Function (DEPDF)

But let us take a "not-necessarily-thermal" look at the BAZ double-exponential equation. In such a situation the probability-of-non-failure reflects, first, the role of the ratio of the element's (material's, system's, human) bearing capacity (analogous to the "activation energy") to the external loading (analogous to the "thermal energy"). When applied to an individual human, the function (1) reflects the role of the ratio of his/hers human-capacity-factor (HCF) (which is analogous to the "activation energy" that characterizes the "bearing capacity" of a system) to the mental/cognitive workload (MWL) (analogous to "thermal energy" that characterizes the level of the loading on the system). In the recently suggested "probabilistic Fitts' law" [27, 28] the probability

$$P = \exp \left[-\lambda t \exp \left(-\frac{W}{2D} \right) \right] \quad (2)$$

of non-failure, i.e., the probability of hitting the target - the black rectangular on the computer screen, increases with the increase in the width of this rectangular (which is analogous to the "activation energy", the "bearing capacity" of the target) and decreases with an increase in the distance from the user to the computer screen (which is analogous to "thermal energy", the "loading"). As to a particular individual's "human quality", the human capacity factor (HCF), the great Russian writer Leo Tolstoy, "God's elder brother", made a rather broad statement about the human quality regardless of a particular situation or application: "A man is like a fraction whose numerator is what he is and the denominator is what he thinks of himself, the larger the denominator - the smaller the fraction" [51]. Thus, the above double-exponential-probability-distribution-function (DEPDF) is a general, flexible, broad, physically meaningful and useful probabilistic quantitative description that is applicable to many physical, "human-in-the-loop" (HITL), ergonomics, medical and clinical systems.

EXAMPLES

Predicted Probability of a Maritime Mission Success and Safety: Application of Weibull Distribution

Let, e.g., a particular aerospace mission of interest consists of n segments (characterized by different probabilities, q_i , of occurrence of a particular harsh environment or some other extraordinary conditions during the fulfillment of the mission/flight at the i -th segment. The segments are characterized also by different durations, T_i , and different failure rates λ_i^e of the equipment and instrumentation. These rates may or may not depend on the environmental conditions but could be affected by aging/degradation and other time-dependent causes. In the simplified example below, we assume that the combined input of the hardware and the software, as far as the performance of the equipment and instrumentation is concerned, is evaluated beforehand and is adequately reflected by the appropriate available failure rate λ_i^e values. These could be either determined from the vendor specifications or, preferably, obtained based on

specially designed and conducted failure oriented accelerated testing (FOAT). FOAT should be preferably geared to a particular predictive model, such as, e.g., BAZ model. Let the probability of the equipment non-failure at the moment t_i of time during the fulfillment of the mission on the i -th segment, assuming that Weibull distribution is applicable, be

$$P_i^e = \exp \left[-\left(\lambda_i^e t_i \right)^{\beta_i^e} \right] \quad (3)$$

where $0 \leq t_i \leq T_i$ is an arbitrary moment of time within the i -th segment, and β_i^e is the shape parameter in the Weibull distribution. One could assume that the time-dependent probability of human non-failure can be represented in the form of Weibull distribution

$$P_i^h(t_i) = P_i^h(0) \exp \left[-\left(\lambda_i^h t_i \right)^{\beta_i^h} \right] \quad (4)$$

where λ_i^h is the failure rate, β_i^h is the shape parameter and $P_i^h(0)$ is the probability of the human non-failure at the initial moment of time $t_i=0$ of the given segment. When $t_i \rightarrow \infty$ the probability of non-failure (say, because of human fatigue or other causes) tends to zero. The probability $P_i^h(0)$ can be assumed particularly in the form of the distribution (1). In an approximate analysis the probability of the mission failure at the i -th segment can be found as (in a rigorous analysis conditional probabilities should be considered)

$$Q_i(t_i) = 1 - P_i^e(t_i)P_i^h(t_i) \quad (5)$$

and the overall probability of the mission failure is

$$Q = \sum_{i=1}^n q_i Q_i(t_i) = 1 - \sum_{i=1}^n q_i P_i^e(t_i) P_i^h(t_i) \quad (6)$$

This formula can be used also for specifying the failure rates and the HCF in such a way that the overall probability of failure would be adequate for the given mission. The assessments based on the formula (6) can be used to choose, if possible, an alternative route or time, so that the set of the probabilities q_i of encounter the environmental conditions of the given severity brings the overall probability of the mission failure to an acceptable low level.

Let, for instance, the duration of a particular vehicular mission be 24 hours, and the vehicle spends equal times at each of the 6 segments (so that $t_i = 4$ hours at the end of each segment), the failure rates of the equipment and the human performance are independent of the environmental conditions and are $\lambda = 8 \times 10^{-4}$ 1/hour, the shape parameter in the Weibull distribution in both cases is $\beta = 2$ (Rayleigh distribution is applicable), the HCF ratio $\frac{F^2}{F_0^2}$ is $\frac{F^2}{F_0^2} = 8$ (so that $\frac{F}{F_0} = 2.828$), the probability of human non-

failure at ordinary conditions is $P_0 = 0.9900$, and the MWL G_i^2/G_0^2 ratios are 1, 2, 3, 4, 5, and occur with the probabilities $q_i = 0.9530, 0.0399, 0.0050, 0.0010, 0.0006$ and 0.0005 . These data indicate that about 95% of the mission time takes place in ordinary conditions. The calculated $\bar{P}_i = \frac{P_i^h(t_i)}{P_i^h(0)}$ ratios for the six

segments of the mission are 1.0000, 0.9991, 0.9982, 0.9978, 0.9964 and 0.9955, and the computed probabilities P_i^h of the human non-failures are 0.9900, 0.9891, 0.9882, 0.9878, 0.9864 and 0.9855. The products $P_i^e P_i^h$ of the equipment and the human non-failures are 0.9900, 0.9891, 0.9882, 0.9878, 0.9864 and

0.9855, and the products $q_i P_i^e P_i^h$ are 0.9435, 0.0395, 0.0049, 0.0010, 0.0006, and 0.0005. Then the predicted probability of the mission's non-failure is $P = \sum_{i=1}^n q_i P_i^e(t_i) P_i^h(t_i) = 0.9900$, and the probability of failure is therefore only $Q = 0.01 = 1\%$. In connection with this result we would like to indicate that when establishing whether this probability is acceptable as an adequate risk level, one should consider also the consequences of a particular failure. Such an effort is, however, beyond the scope of this analysis.

The Current State of Navigator's Health and Its Effect on the Likelihood of Making a Human Error

The current (temporary) state of human health or mind during the fulfillment of an aerospace mission or when encountering an extraordinary situation could affect his/her failure-free performance resulting in what is known as human error. Let us address, as a suitable example of an analogy-based medical application of the PPM, probabilistic human-system-interaction/integration (HSI) in aerospace engineering, or, specifically, navigator's (aircraft pilot's or astronaut's) performance vs. his/hers human-capacity-factor (HCF), with an emphasis is on his/her state-of-health (SoH). The following double-exponential-probability distribution function (DEPDF) for the probability of human-non-failure could be assumed in the form [18]:

$$P^h(F, G, S_*) = P_0 \exp \left[\left(1 - \gamma_s S_* t - \frac{G^2}{G_0^2} \right) \exp \left(1 - \gamma_T T_* - \frac{F^2}{F_0^2} \right) \right] \quad (7)$$

This function enables evaluating the impact of three major factors, namely, the mental workload (MWL) G , the human capacity factor (HCF) F , and the time t (possibly affecting the navigator's performance, such as, e.g., the likelihood of making a mistake, and sometimes even affecting his/her health), on the probability (7) of non-failure. Here P_0 is the initial probability ($t = 0$) of non-failure at a normal (sufficiently low) level of the MWL ($G = G_0$), S_* is the threshold (acceptable level) of the (supposedly continuously monitored/measured, cumulative, effective, indicative, and possibly even multi-parametric) health ("medical") characteristic, such as, say, body temperature, arterial blood pressure, oxyhaemo-metric determination of the level of saturation of blood hemoglobin with oxygen, electrocardiogram measurements, pulse frequency and fullness, frequency of respiration, measurement of skin resistance that reflects skin covering with sweat, etc. etc. (since the time t and the threshold S_* enter the above governing expression as a product $S_* t$, each of these parameters has a similar cumulative impact on the sought probability), γ_s is the sensitivity factor for the symptom S_* ; $G \geq G_0$ is the actual (elevated, off-normal, extraordinary, possibly even time-dependent) MWL, G_0 is the MWL at ordinary (normal) operation conditions, T_* is the mean time to error/failure (MTTF), γ_T is the sensitivity factor for this time, $F \geq F_0$ is the actual (could be off-normal) HCF exhibited or required in a particular condition/situation of importance, F_0 is the most likely (normal, specified, ordinary) HCF. There is a certain overlap, of course, between the levels of the HCF F and the MTTF T_* values: both have to do with the human quality and performance. The difference is, however, that T_* is a short-term characteristic of the navigator's performance that might be affected, first of all, by his/her personality and vulnerability to various influences, while the HCF is a long-term characteristic, such as his/her age, education, experience, ability to think and

act independently and under pressure, and, if necessary, as a team player, etc. etc.

The MTTF T_* might be determined for the given individual by using a highly focused failure-oriented-accelerated-testing (FOAT) on a flight simulator [14], whatever the appropriate definition of failure in such testing might be, while the HCF F , which should also be quantified, cannot obviously be evaluated experimentally and should be quantified using a specially designed methodology. It is noteworthy also that while the P_0 value is defined as the probability of the human-non-failure at a very low MWL level G it could be determined and evaluated also as the probability of the human-non-failure for a hypothetical situation, when the HCF F is extraordinarily high, i.e., for a navigator who is exceptionally highly qualified (like, say, Captain "Sully" in the famous "miracle-on-the-Hudson" event [10]), while the MWL G is still finite, and so is the operation time t . The suggested governing DEPDF function has a nice symmetric form. Indeed, it reflects the roles of the "objective", "external", MWL plus the state-of-health (SoH) impact $E = \left(1 - \gamma_s S_* t - \frac{G^2}{G_0^2} \right)$, as well as of the "subjective", "internal", human capacity factor (HCF) plus the likelihood of human error HCF+HE impacts $I = \left(1 - \gamma_T T_* - \frac{F^2}{F_0^2} \right)$. Here is the rationale below the structures of these expressions. The level of the MWL could be affected by the human's SoH: the navigator might experience a higher MWL, which is not only different for different individuals, but might be quite different for the same individual, depending on his/hers current, short-term, SoH, while his/hers HCF, although could also be influenced by the state of his/her SoH, affects the probability of the human non-failure (HnF) indirectly. In our approach the impact of the human's state-of-health (SoH) could be measured/quantified by the navigator's mean-time-to-error (MTTE), since the human error (HE) is, in effect, a failure, interruption, in his/hers otherwise error-free performance process, is it not? When the human's qualification is high, the likelihood of an error is most likely low, regardless of how harsh the external conditions are. Thus, in our model the "external" factor $E = \text{MWL} + \text{SoH}$ (mental workload plus state-of-health) is a short-term characteristic of the human performance, while the "internal" factor $I = \text{HCF} + \text{HE}$ (human capacity factor plus propensity to make an error) is a more permanent, a long-term characteristic of the navigator's HCF. It is also noteworthy that the human's mind (reflected by his/her MWL) and his/her body's SoH are intricately linked, that such a link is different for different individuals, and that is at present far from being clearly understood and well defined. The suggested formalism is, of course, just a possible and a highly tentative way to account for such a link. Difficulties may arise on some occasions when the MWL and the SoH factors overlap. It is anticipated therefore that the MWL impact in the suggested formalism considers, to an extent possible, various more or less most important influences other than the direct SoH related ones.

Human capacity factor (HCF), unlike mental/cognitive workload (MWL), is a new notion in ergonomics engineering (see, e.g.,) [12-14]. HCF plays with respect to the MWL the same role as strength/capacity plays with respect to stress/demand in structural analysis and in some economics problems. HCF includes, but might not be limited to, the following major qualities that would enable a professional human to successfully cope with an elevated off-normal MWL: age,

fitness, health, personality type, psychological suitability for a particular task, professional experience and qualifications, education, both special and general, relevant capabilities and skills, level, quality and timeliness of training, performance sustainability (consistency, predictability), independent thinking and independent acting, when necessary, ability to concentrate, awareness and ability to anticipate, ability to withstand fatigue, self-control and ability to act in cold blood in hazardous and even life threatening situations, mature (realistic) thinking, ability to operate effectively under pressure, and particularly under time pressure, leadership ability, ability to operate effectively, when necessary, in a tireless fashion, for a long period of time (tolerance to stress), ability to act effectively under time pressure and make well substantiated decisions in a short period of time and in an uncertain environmental conditions, team-player attitude, when necessary, swiftness in reaction, when necessary, adequate trust (in humans, technologies, equipment), ability to maintain the optimal level of physiological arousal. These and other qualities are certainly of different importance in different human-in-the-loop (HITL) situations. It is clear also that different individuals possess these qualities to different degrees. Long-term HCF could be time dependent. To produce suitable figures-of-merit (FoM) for the HCF, one could rank, similarly to the MWL estimates, the above and perhaps other qualities on the scale from, say, one to ten, and calculate the average FoM for each individual and task. Clearly, MWL and HCF measurements should use the same units, which could be particularly non-dimensional. Special psychological tests might be necessary to develop and conduct to establish the level of these qualities for the individuals of significance.

In connection with the approach taken it is noteworthy also that not every model needs prior or even posterior experimental validation. In the author's view, the structure of our governing models does not. Just the opposite: this model should be used as the basis of the FOAT to establish the MWL, HCF, and various human errors (HE) through the corresponding observed and recorded MTTF and his/hers SoH at normal operation conditions and for a navigator with regular skills and of ordinary human capacity. These experiments could be conducted, e.g., on flight simulators, and using various specially developed testing methodologies. Being a probabilistic, not a statistical model, the approach should be used to obtain, interpret, and accumulate relevant statistical information. Starting with collecting statistics first seems to be a time consuming and overly expensive path, often leading to nowhere.

CONCLUSION

While some kind of predictive modeling should always be conducted prior to and, when possible and appropriate, also during accelerated reliability testing, analytical ("mathematical"), preferably probabilistic, modeling, such as the one based on mathematical analogies, should complement computer simulations. Computer simulations and analytical ("mathematical") modeling are based on different assumptions and use different calculation techniques, and if the results obtained using these two major modeling tools are in agreement, then there is a good reason to believe that the data obtained are sufficiently accurate, adequate and, hence, trustworthy. Future work should consider other suitable applications of the addressed "suitable analogy" based approach and a methodology for establishing the ultimate risks, bearing in mind that the

levels of such risks should consider not only the probabilities of the anticipated critical failures, but the consequences of these failures as well.

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